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# TIDAL EVOLUTION OF THE PLANETARY SYSTEM AROUND HD 83443

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## ABSTRACT

Two planets with orbital period ratio approximately 10:1 have been discovered around the star HD 83443. The inner and more massive planet, HD 83443*b*, has the smallest semi-major axis among all currently known exoplanets. Unlike other short period exoplanets, it maintains a substantial orbital eccentricity, $e_1 = 0.079 \pm 0.008$, in spite of efficient tidal damping. This is a consequence of its secular interactions with HD 83443*c* whose orbital eccentricity $e_2 = 0.42 \pm 0.06$. Dissipation, associated with tides the star raises in the inner planet, removes energy but not angular momentum from its orbit, while secular interactions transfer angular momentum but not energy from the inner to the outer planet's orbit. The outward transfer of angular momentum decreases the tidal decay rate of the inner planet's orbital eccentricity while increasing that of the outer planet. The alignment of the apsides of the planets' orbits is another consequence of tidal and secular interactions. In this state the ratio of their orbital eccentricities, $e_1/e_2$, depends upon the secular perturbations the planets exert on each other and on additional perturbations that enhance the inner planet's precession rate. Tidal and rotational distortions of the inner planet along with general relativity provide the most important of these extra precessional perturbations, each of which acts to reduce $e_1/e_2$. Provided the planets' orbits are coplanar, the observed eccentricity ratio uniquely relates $\sin i$ and $C \equiv (k_2/k_{2J})(R_1/R_J)^5$, where the tidal Love number, $k_2$, and radius, $R_1$, of the inner planet are scaled by their Jovian equivalents.

*Subject headings:* celestial mechanics – (stars:) binaries: spectroscopic – (stars:) planetary systems

## 1. INTRODUCTION

Parameters of the HD 83443 planetary system as given by Mayor et al. (2000) are listed in Table 1. The substantial eccentricity of the inner planet's orbit, $e_1 = 0.079 \pm 0.008$, is surprising since other exoplanets with comparable semi-major axes have circular orbits. In the absence of other interactions, tidal dissipation would erode $e_1$ as $d \ln e_1/dt = -1/\tau$ with (Goldreich 1963, Hut 1981),

$$\tau \approx \frac{2}{27}\frac{Q}{k_2}\left(\frac{a_1^3}{GM_*}\right)^{1/2}\left(\frac{m_1}{M_*}\right)\left(\frac{a_1}{R_1}\right)^5. \tag{1}$$

Adopting values appropriate for Jupiter: radius $R_1 \approx 7 \times 10^4$ km, mass $m_1 \approx 2 \times 10^{30}$g, tidal Love number $k_2 \approx 0.5$ (Yoder 1995), and tidal quality factor, $Q \sim 3\times10^5$ (Goldreich & Soter 1966), yields $\tau \sim 3 \times 10^8$ yr. This is in striking contrast to the old age of the star. Isochrone fitting suggests an age of about 6.5 Gyr (N. Murray, private communication), which is consistent with the absence of chromospheric activity and the small projected rotational velocity, $v \sin i < 2$ km/ s.

Secular interactions with HD 83443*c* are undoubtedly responsible for the persistence of HD 83443*b*'s orbital eccentricity (Mayor et al. 2000). Results from calculations by Lasker and Benz displayed in Mayor et al. (2000) show that secular interactions between the inner and outer planets induce anti-correlated oscillations in the two planets' orbital eccentricities. The anti-correlation results because secular interactions transfer angular momentum, $J$, but not energy, $E$, and $e$ is related to $E$ and $J$ by

$$e^2 = 1 + \frac{2J^2E}{m^3(GM)^2}. \tag{2}$$

Similar phenomena are noted in the hierarchical triple star systems HD 109648 (Jha et al. 2000), HD 284163 (Griffin & Gunn 1981), and Algol (see, e.g., Kiseleva et al. 1998).

We argue that eccentricity oscillations were tidally damped a long time ago, and that currently the planets maintain apse alignment (§3). In this state the orbital eccentricity of the inner planet is proportional to that of the outer planet, and both are steady on the secular time-scale. On the much longer tidal time-scale, the individual eccentricities decay, with the decay rate being faster for the inner planet.

We utilize the properties of this system to constrain parameters that are not directly measurable. From the ratio of the two eccentricities, we place bounds on the inclination angle, $i$, of the planets' orbit plane and on the radius of the inner planet.

## 2. CONSEQUENCES OF APSE ALIGNMENT

### 2.1. *Assumptions*

We study secular interactions in $HD83443$'s planetary system under the assumptions that the planets share a common orbital plane and that their apsides remain aligned. The former assumption seems reasonable for planets formed in a gaseous disk. The latter assumption is consistent with the currently observed approximate apse alignment (see Table 1) and is justified by the analysis in

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TABLE 1
PARAMETERS FOR THE TWO PLANETS OF HD 83443.[a]

| Planet | P (d) | $e$ | $\varpi(^\circ)$ | $a$ (AU) | $m\sin i$ |
|---|---|---|---|---|---|
| HD83443$b$ | $2.9853 \pm 0.0009$ | $0.079 \pm 0.008$ | $300 \pm 17$ | 0.0376 | 1.14 |
| HD83443$c$ | $29.83 \pm 0.18$ | $0.42 \pm 0.06$ | $337 \pm 10$ | 0.175 | 0.53 |

[a]Data taken from Mayor et al. (2000). Planet masses are in unit of Saturn mass $M_{\rm Sat} \approx 5.68 \times 10^{29}$g. The host star $M_* = 0.79 M_\odot$.

§3. These assumptions allow us to infer parameters of the planetary system from the observed eccentricity ratio.

### 2.2. *Secular Dynamics*

Mutual secular perturbations of the planets' orbits are calculated following a method devised by Gauss (1818) as outlined in the appendix. Equation (A3) describes the resulting periapse and eccentricity variations. However, as a result of inner planet's proximity to the star, it is subject to additional precessional perturbations of which the most important arise from its related tidal and rotational distortions (Sterne 1939)[3], and from general relativity (Einstein 1916). Expressions for these precession rates are,

$$\left.\frac{d\varpi_1}{dt}\right|_{\rm tide} = 7.5 n_1 k_2 \frac{1+\frac{3}{2}e_1^2+\frac{1}{8}e_1^4}{(1-e_1^2)^5}\frac{M_*}{m_1}\left(\frac{R_1}{a_1}\right)^5, \quad (3a)$$

$$\left.\frac{d\varpi_1}{dt}\right|_{J_2} = 0.5 n_1 \frac{k_2}{(1-e_1^2)^2}\left(\frac{\Omega_1}{n_1}\right)^2\frac{M_*}{m_1}\left(\frac{R_1}{a_1}\right)^5, \quad (3b)$$

$$\left.\frac{d\varpi_1}{dt}\right|_{\rm GR} = 3 n_1 \frac{GM_*}{a_1 c^2 (1-e_1^2)}. \quad (3c)$$

Note that in equation (3b) the planet's spin rate, $\Omega_1$, is scaled by its mean motion, $n_1$, since we expect that tidal dissipation has produced approximate spin-orbit synchronization. We neglect smaller precessional contributions due to the rotational oblateness of the star and to the tidal distortion of the star by the planet.

Numerically, the total rate of extra precessions amounts to,

$$\begin{aligned}\Delta &\equiv \left.\frac{d\varpi_1}{dt}\right|_{\rm tide} + \left.\frac{d\varpi_1}{dt}\right|_{J_2} + \left.\frac{d\varpi_1}{dt}\right|_{\rm GR} \\ &= (7.79\, C \sin i + 1.52) \times 10^{-11}\,{\rm s}^{-1}, \quad (4)\end{aligned}$$

where we set $\Omega_1 = n_1$ and introduce the dimensionless number $C \equiv (k_2/k_{2J})(R_1/R_J)^5$, with $R_J$ and $k_{2J}$ assigned values appropriate for Jupiter. Observations determine $k_{2J} = 0.494$, close to that of an $n = 1$ polytrope, $k_2 \approx 0.51$, whereas Saturn has a smaller Love number, $k_2 = 0.317$ (Yoder 1995) because it is more centrally condensed as a result of possessing a relatively larger core of heavy elements.

We incorporate the additional precessional terms into the secular perturbation equations (see eq. [A1]). In a state of apse alignment, there is no secular exchange of angular momentum between the planets. Therefore, their orbital eccentricities are constant on the secular timescale. Moreover, the eccentricity ratio, $e_1/e_2$, is a monotonically decreasing function of $\Delta$. To match the observationally determined value of $e_1/e_2 = 0.19$ requires $\Delta \approx 9.1 \times 10^{-11} \sin^{-1} i\ {\rm s}^{-1}$. This value is greater than the precession rate due to general relativity and provides evidence for the tidal and rotational distortion of the inner planet. Because the inner planet is not observed to transit, $\sin i < 0.99$, which implies $C \geq 0.9$. Nearly pole-on orbits, $\sin i < 0.20$, are excluded if we accept that $R_1 < 2R_J$, a conservative upper-limit as judged from Fig. 1 of Burrows et al. 2000. This implies $m_1 < 5.45 M_{\rm sat}$. Figure 1 shows $e_1$ as a function of $C$ for different values of $\sin i$ with $e_2$ fixed at its observed value.

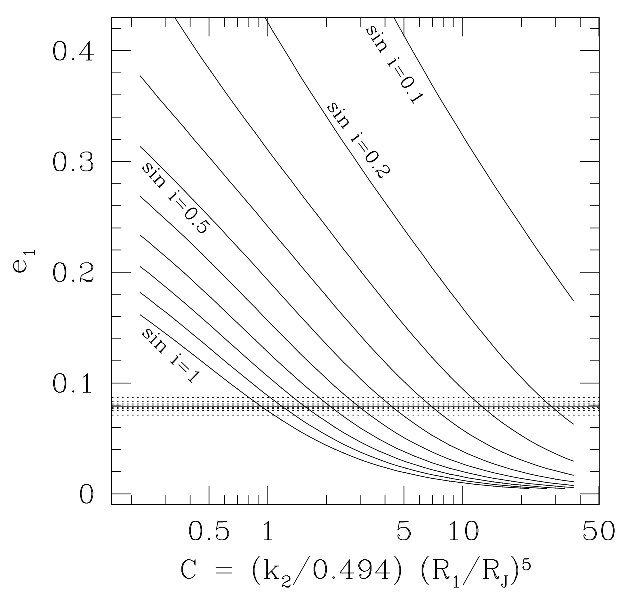


FIG. 1.— Values of the inner planet eccentricity, $e_1$, as a function of $C$ for different values of $\sin i$. The ten solid lines are for $\sin i$ varying in equal increments from 0.1 to 1.0. The horizontal band is centered on the measured value of $e_1$ and its vertical width spans a $1-\sigma$ error bar in each direction. Relevant parameters are taken from Table 1.

### 2.3. *Tidal Dynamics*

[3]The symbol $k_2$ also appears as the apsidal motion constant in stellar literature in which case it is a factor 2 smaller than the Love number.

Tidal dissipation in the approximately synchronously rotating inner planet decreases its orbital energy without significantly changing its orbital angular momentum. It follows from equations (1) and (2) that this causes $a_1$ to vary as $d\ln a_1/dt \approx -2e_1^2/\tau$. Since secular interactions transfer angular momentum but not energy between the planets' orbits, $a_2$ remains constant. Conservation of total angular momentum leads to,

$$\begin{aligned}\frac{dJ_1}{dt} &= J_1\left(\frac{1}{2}\frac{d\ln a_1}{dt} - \frac{e_1^2}{1-e_1^2}\frac{d\ln e_1}{dt}\right)\\ &= -\frac{dJ_2}{dt} = J_2\,\frac{e_2^2}{1-e_2^2}\frac{d\ln e_2}{dt},\end{aligned} \tag{5}$$

where $J_i = \sqrt{1-e_i^2}m_i n_i a_i^2$. Apse alignment implies $e_1 = f(a_1, e_2)$, from which we deduce that

$$\begin{aligned}\frac{d\ln e_1}{dt} &= -\frac{1}{\tau}\,\frac{f_{e_2}(1-e_1^2)+2e_2^2 f_{a_1}\frac{J_2(1-e_1^2)}{J_1(1-e_2^2)}}{f_{e_2}+\frac{J_2}{J_1}\,(\frac{1-e_1^2}{1-e_2^2})\,(\frac{e_2}{e_1})^2},\\ \frac{d\ln e_2}{dt} &= -\frac{1}{\tau}\,\frac{(1-e_1^2)-2e_1^2\,f_{a_1}}{f_{e_2}+\frac{J_2}{J_1}\,(\frac{1-e_1^2}{1-e_2^2})\,(\frac{e_2}{e_1})^2},\end{aligned} \tag{6}$$

where $f_{a_1} \equiv \partial\ln f/\partial\ln a_1$ and $f_{e_2} \equiv \partial\ln f/\partial\ln e_2$. Numerically, $f_{a_1} \approx 8$ and $f_{e_2} \approx 1$, so currently $e_1$ and $e_2$ are damping on time-scales of $8.6\tau$ and $40\tau$, respectively; $e_1$ damps faster than $e_2$ because the inner planet's semi-major axis is shrinking.

## 3. LINEAR APPROXIMATION

In §2 we assumed that tidal dissipation had delivered the planets' orbits into a state of periapse alignment. Here we justify this assumption by studying the secular dynamics and tidal evolution under the somewhat inaccurate approximation of small orbital eccentricities.

Our starting point is the expansion of the secular disturbing function to second order in $e_1$ and $e_2$. Substituting the expanded disturbing function into Lagrange's equations for the variations of the $e$'s and the $\varpi$'s, and adding a term to represent tidal damping of $e_1$, yields the following set of linear equations (MD2000)

$$\frac{dI_1}{dt} = iA_{11}I_1 + iA_{12}I_2 - I_1/\tau, \qquad \frac{dI_2}{dt} = iA_{21}I_1 + iA_{22}I_2, \tag{7}$$

for the complex variable $I_i \equiv e_i\exp(i\varpi_i)$. The coefficients $A_{ij}$ read:

$$\begin{aligned}A_{11} &= \frac{\alpha^2}{4}\frac{m_2}{M_*}n_1B_1 + \Delta, \quad A_{12} = -\frac{\alpha^2}{4}\frac{m_2}{M_*}n_1B_2,\\ A_{21} &= -\frac{\alpha}{4}\frac{m_1}{M_*}n_2B_2, \qquad A_{22} = \frac{\alpha}{4}\frac{m_1}{M_*}n_2B_1,\end{aligned} \tag{8}$$

with $\alpha = a_1/a_2$, $B_1 = b_{3/2}^{(1)}$, and $B_2 = b_{3/2}^{(2)}$; $b_s^{(j)}$ is the usual Laplace coefficient. The symbols $\Delta$ and $\tau$ retain their definitions from §2.

In the absence of tidal damping ($\tau \to \infty$), the general solution of equation (7) can be expressed as a super-position of two eigenvectors $\mathcal{I}_*$ with

$$\mathcal{I}_* = [e_{1*}\exp(i\,\phi_{1*}), e_{2*}\exp(i\,\phi_{2*})]\,\exp(i\,g_*t), \tag{9}$$

where

$$\begin{aligned}g_* &= \frac{(A_{11}+A_{22}) \pm \sqrt{(A_{11}-A_{22})^2+4A_{12}A_{21}}}{2}, &\text{(10a)}\\ \frac{e_{1*}}{e_{2*}} &= \left|\frac{g_*-A_{22}}{A_{21}}\right| = \left|\frac{A_{12}}{A_{11}-g_*}\right|, &\text{(10b)}\\ \phi_{1*} &= \phi_{2*} + \frac{\pi}{2}\,[1+\mathrm{sgn}(g_*-A_{22})]\,. &\text{(10c)}\end{aligned}$$

We distinguish the two eigensolutions by $* = p$ and $* = m$ according to the plus or minus sign that appears before the square root in the expression for $g_*$. Since $(e_{1p}/e_{2p})\,(e_{1m}/e_{2m}) = A_{12}/A_{21} \approx 1.0$, one solution satisfies $e_1 < e_2$ and the other $e_1 > e_2$. If we take $\Delta = 5\times10^{-11}\,\mathrm{s}^{-1}$, the $* = m$ solution reproduces the observed eccentricity ratio $e_{1m}/e_{2m} = 0.19$ and apse alignment.[4] The other solution has $e_{1p}/e_{2p} = 5.26$ and anti-aligned apses. If both eigenvectors have non-zero lengths, the orbital eccentricities oscillate out of phase on time-scales of thousands of years ($\sim 1/g_*$) as depicted in Mayor et al. (2000).

When $\tau$ is finite, $a_1$ and therefore $A_{ij}$ vary with time. Since $\tau \gg 1/g_*$, the solution for the eigenvectors given by equation (10) remains valid. However, the eigenvector components, $e_{i*}$, decay slowly with time as

$$\begin{aligned}\gamma_{1*} &= \frac{1}{\tau}\,\frac{1+2e_{2*}^2\,f_{a_1}\frac{J_2}{J_1}}{1+\frac{J_2}{J_1}\,(\frac{e_{2*}}{e_{1*}})^2},\\ \gamma_{2*} &= \frac{1}{\tau}\,\frac{1-2e_{1*}^2\,f_{a_1}}{1+\frac{J_2}{J_1}\,(\frac{e_{2*}}{e_{1*}})^2},\end{aligned} \tag{11}$$

where $\gamma_{i*} \equiv -d\ln e_{i*}/dt$. In the limit of small eccentricity, the expressions for the nonlinear damping rates given by equation (6) reduce to those in equation (11) provided $f_{e_2} = 1$ as is appropriate for the linear problem. Each of the four eigenvector components has its individual damping rate. Both components of the $p$ eigenvector decay more rapidly than those of the $m$ eigenvector. If the current state consists mostly the $m$ eigenvector with a small admixture of the $p$ eigenvector, both components in the latter would decay on time-scale $\approx \tau$, while the dominant components $e_{1m}$ and $e_{2m}$ would decay much more slowly, as $\gamma_{1m}^{-1} \approx 7\tau$ and $\gamma_{2m}^{-1} \approx 32\tau$.

It is notable that the asymmetry in damping rates between the aligned and the anti-aligned solutions only applies if the inner planet has an orbital period $\leq 5$ days. For longer periods, the additional precession rates (eqs. [3a] - [3c]) are unimportant and $(e_{1m}/e_{2m}) \sim (e_{1p}/e_{2p}) \sim 1$.

## 4. CONCLUSIONS

Both the survival of HD 83443*b*'s orbital eccentricity and the locking of its apse to that of HD 83443*c* are straightforward consequences of secular and tidal dynamics. The ratio $e_1/e_2$ provides compelling evidence for the tidal and rotational distortion of HD 83443*b*. It constrains unknown system parameters by relating $C \equiv (k_2/k_{2J})(R_1/R_J)^5$ to $\sin i$. In particular, we find $C > 0.9$.

[4] This value of $\Delta$ is different from that required in §2.2 because the linear approximation is inaccurate for $e_2 = 0.42$.

Apse alignment indicates that a minimum amount of tidal dissipation has occurred over the life-time of the system, taken to be $N$ Gyrs. By requiring that this is at least 3 times longer than $\tau$, we deduce that $Q < 5\times 10^5\, NC \sin i$. It may also be possible to infer a lower limit for $Q$ by integrating the orbits backward in time.

At the present time $n_1/n_2 = 9.99 \pm 0.06$, suggestive of a $10:1$ mean motion resonance. This is probably just a coincidence. Moreover, no physically important effects would be associated with this resonance should it exist.

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## APPENDIX

### SECULAR PERTURBATIONS

Secular changes of the inner planet's periapse angle, $\varpi_1$, and eccentricity, $e_1$, are described by

$$\frac{de_1}{dt} = \frac{de_1}{dt}\Big|_2, \qquad \frac{d\varpi_1}{dt} = \frac{d\varpi_1}{dt}\Big|_2 + \frac{d\varpi_1}{dt}\Big|_{\rm tide} + \frac{d\varpi_1}{dt}\Big|_{J_2} + \frac{d\varpi_1}{dt}\Big|_{\rm GR}, \tag{A1}$$

where the subscript 2 denotes effects arising from secular interactions with the outer planet. Similar equations apply to the evolution of $\varpi_2$ and $e_2$ but the only significant perturbations are those from the inner planet. Expressions for the precession rates due to the tidal and rotational distortion of the inner planet and to general relativity are given in the main body of the text. We obtain $d\varpi_1/dt|_2$ by applying a method devised by Gauss (1818) as outlined below.

Consider a pair of coplanar rings formed by spreading the mass of each planet along its orbit such that the line density is inversely proportional to the orbital velocity. We decompose into radial, $\mathcal{R}$, and tangential, $\mathcal{T}$, components the gravitational force exerted on a line element of length $r_1 df_1$ by a line element of length $r_2 df_2$, where $f$ denotes true anomaly and $r$ distance from the star;

$$\mathcal{R} = \frac{G\rho_1\rho_2 r_1 r_2 df_1 df_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2}\cos\theta, \qquad \mathcal{T} = \frac{G\rho_1\rho_2 r_1 r_2 df_1 df_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2}\sin\theta, \tag{A2}$$

with $\theta$ the angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2 - \boldsymbol{r}_1$. We remove $\theta$ in favor of $\phi$, the angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ ($\phi = f_2 + \varpi_2 - f_1 - \varpi_1$), by applying the geometric relations, $|\boldsymbol{r}_1 - \boldsymbol{r}_2|\sin\theta = r_2\sin\phi$ and $|\boldsymbol{r}_1 - \boldsymbol{r}_2|\cos\theta = r_2\cos\phi - r_1$. Then we integrate over $df_2$ to obtain the components of the total force due to planet 2 on the mass of planet 1 in the line element $r_1 df_1$. When this result is substituted into Gauss' form of the perturbation equations (Gauss 1818) and integrated over $f_1$, we obtain[5]

$$\begin{aligned}
\frac{d\varpi_1}{dt}\Big|_2 &= \frac{\sqrt{1-e_1^2}}{m_1 n_1 a_1 e_1}\oint df_1\left[-\cos f_1 \oint df_2\,\mathcal{R} + \frac{(2+e_1\cos f_1)\sin f_1}{1+e_1\cos f_1}\oint df_2\,\mathcal{T}\right],\\
\frac{de_1}{dt}\Big|_2 &= \frac{\sqrt{1-e_1^2}}{m_1 n_1 a_1}\oint df_1\left[\sin f_1 \oint df_2\,\mathcal{R} + \frac{(2\cos f_1 + e_1 + e_1\cos^2 f_1)}{1+e_1\cos f_1}\oint df_2\,\mathcal{T}\right].
\end{aligned} \tag{A3}$$

We have verified that when integrating these equations with parameters appropriate to HD 83443's planetary system, we obtain results that agree well with those obtained by direct integration of the orbits using the SWIFT package (Levinson & Duncan 1991).

[5]Our derivation is slightly different from that given in MD2000, but the results are identical.